The Feynman Project

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Introduction

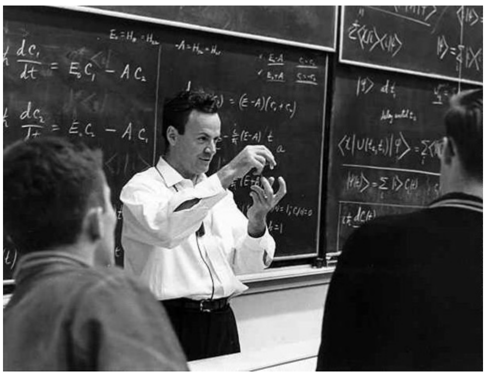
The Feynman point is defined as a point of six identical digits after each other in a number. In π the first Feynman Point is ...1134**999999**83... which occurs after digit number 762. The second Feynman Point in π is ...3865**999999**28... which occurs after digits number 193034. The point is named after the American physicist Richard Feynman, who started a lecture by saying that he would like to memorize the digits of π until that point.

In this paper we will investigate the first- and second Feynman Point in the digits of π and see how 'likely' these are.

In mathematics, a normal number is a real number whose infinite sequence of digits in every base ξ is distributed uniformly in the sense that each of the ξ digit values has the same natural density $1/\xi$. In other words the digits $\{0, 1, ..., 8, 9\}$ occurs equally many times in the number. The number π is believed to be a normal number, however, it has never been proved. In this paper we assume that π is a normal number and most of the calculations is based on this assumption.

We will briefly investigate if π is a normal number and investigate other sequences of the first 10 000 digits of π.

Then we calculate the probabilities that the first - and second Feynman Point occurs in π. Finally I have made a stochastic simulation study of the first - and second Feynman Point to calculate and verify these probabilities. This has successfully been implemented in the program R and all the codes are available.



(Feynman is in the middle.[1])

The discovery of the Feynman Point

The British mathematician William Shanks took 15 years to calculate 707 decimal places of π. It was accomplished in 1873, however, it was only correct up to the 527 places. This error was highlighted in 1944 by D.F. Ferguson using a mechanical desk calculator. Shanks spend his time on calculating mathematical constants. He would calculate new digits all morning, and then he would spend all afternoon checking his morning's work.

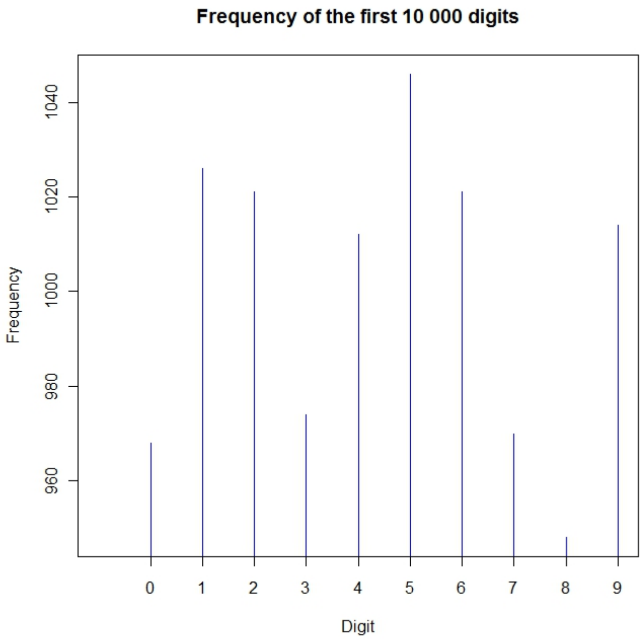
In 1947 D. F. Ferguson calculated π to 808 decimals places. The first Feynman Point was discovered. In 1958 the first 10 000 digits of π were calculated and in 1966 the first 250 000 digits of π were calculated. The second Feynman Point was discovered.

A great Pi Room in the science museum Palais de la Décourverte in Paris was build in 1937. The 707 digits of π were inscribed in the wall in the room. It's a mystery why the inscribed digits weren't changed in 1944 when Ferguson highlighted the error. At the time Paris was occupied by Germany and in 1949 they changed the inscribed digits in the Pi Room. Sadly the Feynman Point wasn't shown in the room.

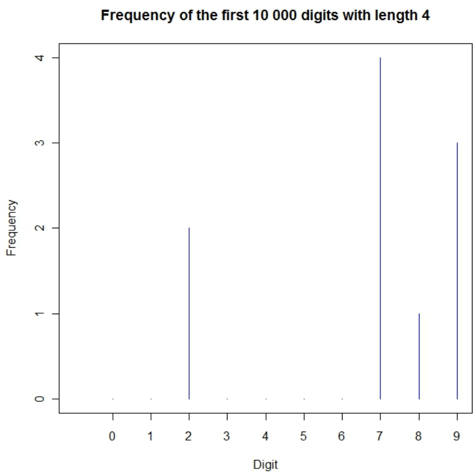
(Mark is in the Pi Room in Paris.[2])

Frequency of the digits

We now want to see the frequency of each digit $0, ..., 9$ in the first 10 000 digits of π. This is shown in the diagram below and the R code can be found in the Appendix under 'R Code A'.



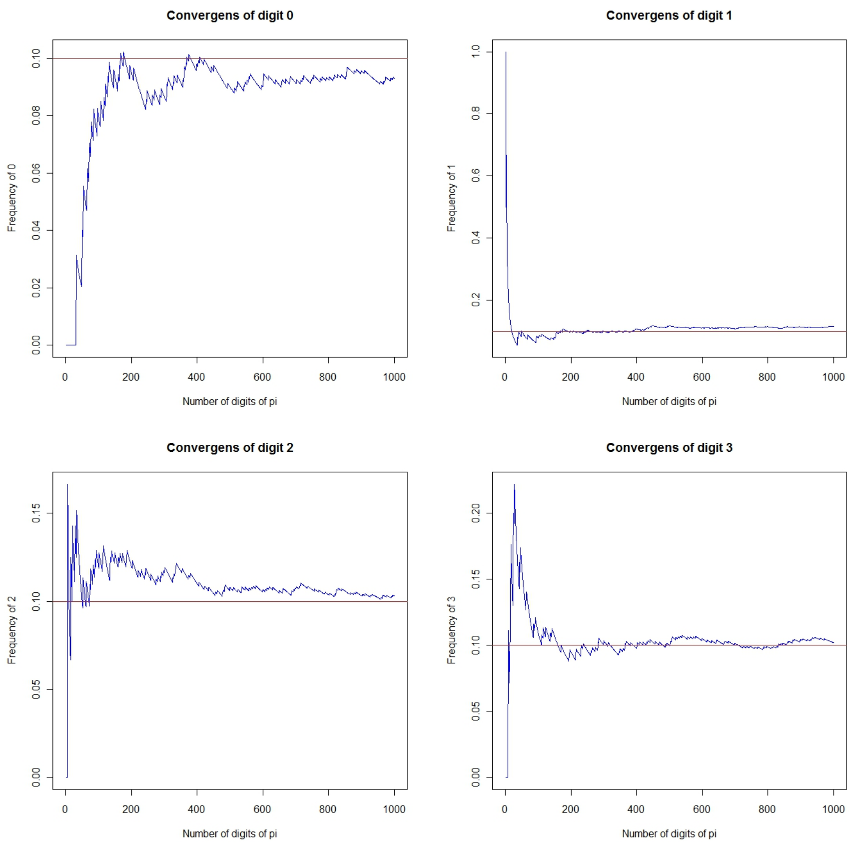
This plot shows the frequency of the sequences $(0000), ..., (9999)$ in the first 10 000 digits of π.

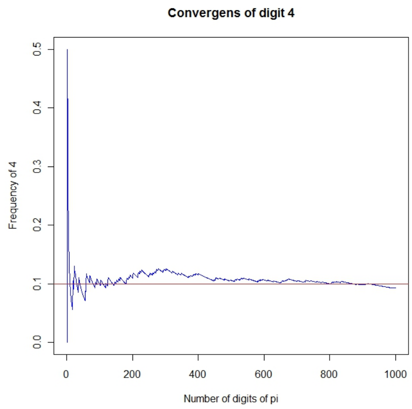


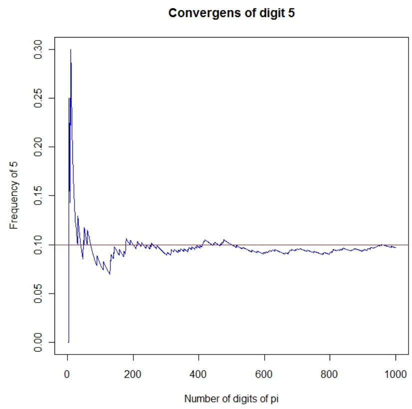
There are no other identical sequence in the first 100000 digits of π than the Feynman point that have length 5. Before the first Feynman Point where are no equally digits with length 4. We can see that 7 is the most common digits of string length 4. This begins after digit number 5322 and is beautiful to witness.

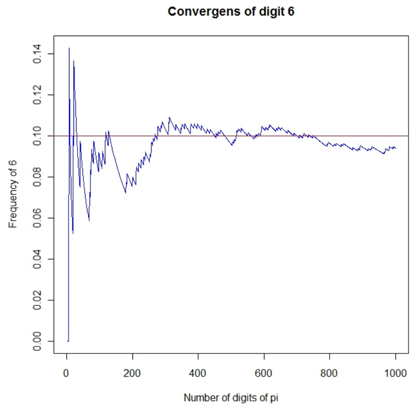
Converges

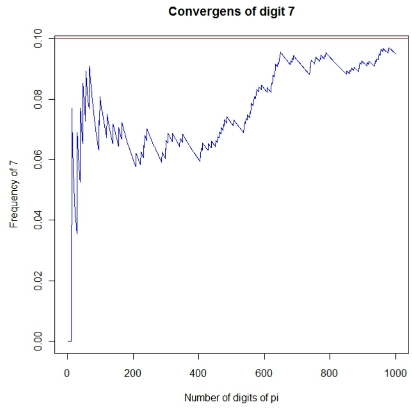
In this section we will briefly show that whether or not π is a normal number. The following plots shows the development of the frequency of each digits. Since the frequency for all the digits is convergent to $\frac{1}{10}$ (the red line) it indicates that π is a normal number. Furthermore we can see that digit 0 and 7 behave strange. There are many digits before digit 0 even occurs, namely 32. This event has probability $(\frac{9}{10})^{31} \approx 0.038$ which is unlikely, however, not as unlikely as the Feynman Point. The R code for the plots can be found in Appendix under 'R Code B'.

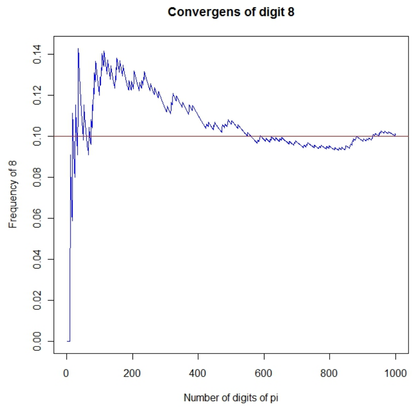


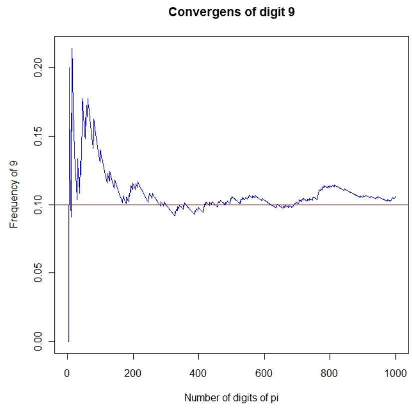












Probabilities of the existence of Feynman Points

We wish to calculate the probability of the existence of a Feynman Point. We can consider this as a binomial distribution where

$$P(\text{Number of succes} = k) = \binom{n}{k}(p)^k(1-p)^{n-k}$$

where n is the number of trials and p is the probability of success on the ith trial, where $i \in \{1, ..., n\}$. We clearly have that $p = 10^{-6}$ and n is the length of the sequence we wish to analyse. With these parameters we have that

$$P(k = 0, n, p) = \binom{n}{0}(10^{-6})^0(1 - 10^{-6})^{n-0} = (1 - 10^{-6})^n$$

is the probability that there are zero Feynman Points in the first n digits of a normal number.

We wish to calculate the probability that there exist at least one Feynman Point after the first 762 digits and then the probability that there exist at least two Feynman Points after the first 193034 digits of π. This have been calculated in R. Notice that we multiply with 10 since we are looking for ten disjoint Feynman Points $(000000, ..., 999999)$.

```
m1=762
m2=193034
# First Feynman Point.
p1=10*(1- pbinom(0,size=m1-6,prob=1/10^6))
p1

# Second Feynman Point.
p2=10*(1- pbinom(1,size=m2-6,prob=1/10^6))
p2
```

The output is $p_1 = 0.007557$ and $p_2 = 0.16397$. Clearly we see that the First Feynman Point is extremely rare to exist already after 762 digits however the existence of two Feynman Point is more likely to exist after 193034 digits.

Another interesting think to investigate is when it's 'likely' that the Feynman Point occurs i.e. we want to find the length n of the sequence when $b(k = 0, n, p = 1 - 10^{-5}) = \frac{1}{2}$. In this case we have

$$n = \frac{\log(1/2)}{\log(1 - 10^{-5})} \approx 69314.$$

In other words, reciting more than 69314 digits it's more likely that there exist a Feynman Point than there doesn't.

Stochastic simulation study of the Feynman Points

In this section we calculate and verify the above probabilities. This is done by stochastic simulation by testing whether or not a Feynman Point exist for a random generated 762-digit number and similarly to test whether or not at least two Feynman Points exist for a generated 193034-digit number.

The first Feynman Point

The result for the first Feynman Point after 100 000 simulations is 0.007558.

```
# Number of simulations.
n=100000

# Number of digits.
m=762

# Generate n random numbers of length m
# and test whether or not the Feynman Point exist.
v=c()
for(k in 1:n){
u=sample(0:9,m,replace=TRUE)
num=function(j){
r=0
for(i in 6:(m-6)){
        if(all(rep(j,6)==c(u[i-5],u[i-4],
        u[i-3],u[i-2],u[i-1],u[i])  ) == TRUE ){
                r=r+1
        }
}
        return(r)
}

# Count the numbers of Feynman Points
# (000000),...,(999999).
o0=num(0)
o=c()
for(j in 1:9){
        o[j]=num(j)
}
o=c(o,o0)
v[k]=sum(o)
}

# Calculate the probability.
prb=sum(v)/n
prb
```

The second Feynman Point

The result for the second Feynman point after 100 000 simulations is 0.16.

```
# Number of simulations
n=100000

# Number of digits
m=193034

v=c()
for(k in 1:n){
u=sample(0:9,m,replace=TRUE)
num=function(j){
r=0
# t counts the number of times at least two
# Feynman Points occurs.
t=0
for(i in 6:(m-6)){
        if(all(rep(j,6)==c(u[i-5],u[i-4],
        u[i-3],u[i-2],u[i-1],u[i])  ) == TRUE ){
                r=r+1
        }
}
        if(r>1) {
        t=t+1
        }
        else{t=0}
        return(t)
}

o0=num(0)
o=c()
for(j in 1:9){
        o[j]=num(j)
}
o=c(o,o0)
v[k]=sum(o)
}


prb=sum(v)/n
prb
```

Conclusion

Our plots indicates that π is a normal number and we calculated the probability of the occurrences of the first two Feynman Point. We can conclude that the first Feynman Point is extremely rare to occur in the first 762 digits of π digits whereas the occurrences of two Feynman Points is more normal to occur after 193034 digits of π. The probability for the first and second Feynman Point was $p_1 = 0.0.007558$ and $p_2 = 0.16$. Our final stochastic simulation study successfully verify these calculated probabilities.

References

[1] Picture of Feynman *www.richard-feynman.net*

[2] Picture of Mark *www.worldpifederation.org*

Appendix

R Code

Diagram

```
p=read.table("C:\\...\\data10000.txt",
header=TRUE)

# Splitting the data into lines
text = readLines("C:\\...\\data10000.txt"
,encoding="UTF-8")

v=c(strsplit(text,"")[[1]])
# convert string v to integer v
strtoi(v, base = 10)

u0=sum(v==0)
u=c()
for(i in 1:9){
        u[i]=sum(v==i)
}
u=c(u0,u)

t=seq(0,9,by=1)
plot(u,col="blue",main="Frequency of the
first 10 000 digits",xlab="Digit",
ylab="Frequency",type="h",xlim=c(0,10),xaxt="n")
axis(1,at=1:10,labels=t[1:10])
```

```
## Higher sequences , namely 4 and 5

ind=function(j){
m=4
r=0
for(i in 1:(length(v)-m) ){
        if(all(c(v[i],v[i+1],v[i+2],v[i+3])==rep(j,m)
            )==TRUE) {
r=r+1
}
}
return(r)
}

o=c()
o0=ind(0)
for(k in 1:9){
        o[k]=   ind(k)
}
o=c(o0,o)

t=seq(0,9,by=1)
plot(o,col="blue",main="Frequency of the first
10 000 digits with length 4",
xlab="Digit",ylab="Frequency",
type="h",xlim=c(0,10),xaxt="n")
axis(1,at=1:10,labels=t[1:10])
```

Convergens

```
p=read.table("C:\\...\\data10000.txt",
header=TRUE)
# Splitting the data into lines
text = readLines("C:\\...\\data10000.txt"
,encoding="UTF-8")

v=c(strsplit(text,"")[[1]])
# convert string v to integer v
strtoi(v, base = 10)

# number of 0
k=0
sum(v==k)/length(v)

#m<length(v)
```

```
del=function(m){
sum(v[1:m]==k)/length(v[1:m])
}

u0=c()
for(i in 1:100){
        u0[i]=del(i)
}
plot(u0,main="Convergens of digit 0",col="blue",
xlab="Number of digits of pi",ylab="Frequency of
   1",type="l")
abline(h=0.1,col="red")
```